



Contents lists available online at Urmia University

Iranian Industrial Economics Studies



Tax competition: Quality and quantity of provision of public goods

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ARTICLE INFO

Article history:

Receive date: 12 March 2018
Revise date: 17 April 2018
Accept date: 30 May 2018

JEL classification:

D00
H20
H41
H70
H71

Keywords:

Marginal cost of public funds
Microeconomic
Provision of public goods
Quality
Samuelson rule
Tax competition

ABSTRACT

The present study was an attempt to investigate local governments' reaction to financing the quality of public goods provision. More especially, whether the local governments must tax the mobile capital or not was addressed. To this end, = Samuelson's rule and the conditions under which the optimal allocation of resources for private and public goods were examined. The findings demonstrated that if local governments finance the public goods by taxing the households without varying their tax rate on capital, the optimality as defined by Samuelson's rule, is constrained by the funding of the quality of public goods. However, taxing the capital modifies the Samuelson's rule. Thus, there is a supplementary cost supported by the households linked to a distorting tax.

1. Introduction

Over the past 15 years, a number of researchers have addressed the issue of tax competition by focusing on the inefficiency occurring in fiscal policies that local governments would like to adopt in order to attract a great number of private investors. Therefore, the difficulty to know the optimal tax rate can lead to local insufficiency of public goods. To maintain a lower tax rate to attract capital, Oates (1972) proved that local governments must provide the amount of local public spending below the level at which the marginal benefits equalize the marginal cost, especially for the expenses that do not directly benefit from private investors. Wildasin (1989) studied fiscal externalities problem of tax policies that local governments create. Since Tiebout's (1956) study, a myriad of researchers (e.g., Wilson, 1986; Zodrow & Mieszkowski, 1986) have investigated tax competition issue in atomizing the number of jurisdictions and considering mobile capital and

immobile labor. Some studies analyzed the impacts of the mobility of labor and capital on tax competition to supply public goods. Bucovetsky and Wilson (1991) concluded that based on the previous models of tax competition, local public goods are sub-optimal. In contrast, tax available by governments is deemed as efficient when both "source-taxation" and "residence-based taxation" taxes are available even in the absence of wage taxation. Other studies examined the effect of different types of taxes to figure out if the non-lump sum taxes have the same influences as the lump sum rates have in cases where local governments use more than two fiscal instruments to finance public goods provision.¹

Gordon (1986) and Bucovetsky and Wilson (1991) confirmed that there would not be a tax competition

1. See Hoyt (1991), Krelove (1993), Burbidge and Myers (1994), Wilson (1995), Wildasin and Wilson (1996).

problem if local governments utilized head taxes or other forms of lump-sum taxation.

Nevertheless, given the current world economic crisis, it is necessary that the affected countries attract capital, increase tax revenues, and use these revenues effectively to finance public goods. Thus, the introduction of the quality factor of public goods can be indicative of attractiveness of investments.¹

This element has been overlooked in the literature on tax competition theory. The present study is an attempt to address this issue through the following question.

Should local governments introduce the quality as the second characterizing factor of public goods?

Local governments supply public goods consisting of a combination of a quantity and a "minimum quality" for economic agents residing in their locality. From this perspective, it should be possible to determine the quality standard upfront to define the quality notion.²

The quality of public goods is influenced when there is a rising demand of taxpayers reluctant to be treated as administered and willing to be treated like customers in a context of economic crisis, or when there is an administration suffering from numerous organizational and operational weaknesses. According to Samuelson (1954), public goods are not only destined to a final consumption, but they also support firm activities (i.e., knowledge, infrastructure, etc.). Public goods are considered to be essential for market transactions (e.g., law of agreement). In this regard yet for different purposes, a number of researchers examined the mechanism of tax competition and its effects in the presence of quality of public services on the localization of capital and households' welfare. Hoyt and Jensen (2001) demonstrated how the differentiation of the quality of education can improve the differential impacts of tax competition and households' welfare. Gabe and Bell (2004) suggested that a local fiscal policy of reduced government spending with decreased public services may attract fewer firms.³

The present study is aimed at investigating the significance of the quality of public goods in the context of tax competition. We explain the existence of some consumer (households') reactions towards

quality based on "the conventional rule of Samuelson (1954)". We use the same assumptions as those of the models proposed by Zodrow and Mieszkowski (1986). These researchers used distorting taxes that affect the optimal allocation of public goods. This distortion of taxes generates the notion of the marginal cost of public funds (MCPF). In fact, there is an additional cost borne by households relating to the use of the distorting taxes. The consequence is that the Samuelson rule of optimality for public-goods is changed.

The organization of this paper is as follows: In Section 2, we discuss different assumptions of the model proposed by Zodrow and Mieszkowski (1986). We extend the existing tax model by including the quality of public goods in the used utility function. In section 3, we describe the social objectives of local governments in terms of supply of public goods. Then, we present a resolution of the model, and various configurations of the provision of public goods are provided. Finally, conclusion is presented in section 4.

2. The model

Many models of tax competition consider a certain number of assumptions based on Zodrow and Mieszkowski's (1986) model that is extended in this study. We consider an economy composed of M identical jurisdictions (i.e., local governments) with $M \geq 2$. Each local government i is inhabited by a set of homogeneous and sedentary representative households (*normalized to unity*). The representative resident possesses all local lands and has a fraction of an available capital stock K_i Capital that is perfectly mobile between local governments without travel costs. The total stock of capital is assumed to be fixed in economy $\sum_{i=1}^M K_i = \bar{K}$. In each jurisdiction, firms use capital to produce their output, and this capital is perfectly mobile between the jurisdictions and some locally fixed factors such as land, which is entirely held by households in each local government. The production technology of a firm i denoted $F_i(K_i)$ occurs through inputs of capital K_i land and a fixed factor. The production function is a decreasing scale return, twice continuously-differentiable, i.e., $(\partial F_i / \partial K_i) > 0$ and $(\partial^2 F_i / \partial K_i^2) < 0$. The capital is mobile and is attracted by the local governments offering the best return after taxation. The arbitrage condition equals the net return of capital in each local government $F_{K_i}(K_i) - t_i = \rho$. ρ refers to the net return of the capital and t_i describes the tax on mobile capital.⁴ Assuming that households of each local government consume a private good C_i , and a quantity of public goods G_i with a quality Q_i , the households' preferences are represented by a utility function $U_i = (C_i, G_i, Q_i)$ where $(\partial U_i / \partial C_i) > 0$, $(\partial U_i / \partial G_i) > 0$ and $\partial U_i / \partial Q_i > 0$ which respectively refer to: the variation of the total utility resulting from the addition of one unit

1. For additional information, see Global Competitiveness Report 2016-2017 published by the World Economic Forum within the framework of the Global Competitiveness and Risks Team.

2. According to Palmer et al. (1991), the quality notion can be viewed as the provision of quality services to satisfy a population while taking into account the technological and resource constraints. For Roemer and Montoya-Aguilar (1989), the quality of a public good is measured by the level at which it meets predefined standards.

3. See the studies conducted by Jud and Watts (1981), Henderson and Thisse (1997), B enassy-Qu er e et al. (2005), Fatica (2010), and Ould Abdessalam et al. (2014).

4. Local government provides a public good that it finances by taxing the mobile capital at a tax rate $t_i \in [0, 1]$.

to the two types of goods C_i , G_i and quality Q_i . The marginal utility is positive, and the total utility increases with the consumed amount of goods $(\partial^2 U_i / \partial C_i^2) < 0$, $(\partial^2 U_i / \partial G_i^2) < 0$, and $(\partial^2 U_i / \partial Q_i^2) < 0$.

After explaining a number of basic assumptions of [Zodrow and Mieszkowsky's \(1986\)](#) model of tax competition, we propose an extension of the model by including the quality of public goods in the light of budget constraint of government i's. We ask here whether determining the scale of quantity of public goods and their quality follows the planning process by local governments. If the quality of a public good is fixed by the local government, then quantity is determined by the local elects. The local governments provide a quantity of public goods G_i with a quality Q_i the households, and it is financed by a tax on mobile capital $t_i K_i$ and lump-sum tax on households H_i at its maximum level $H_i \leq \bar{H}$. The governments establish some standards which define the quality of a public good Q_i , that is, the characteristics that should be taken into account by the transport infrastructures or specific steps to improve the safety of the transport network. For instance, organizations in a public education system are insured by the government and rely on the authorities in the government to develop the public service.¹ The governments require certain expenditures called pedagogical qualities to spend on equipment for computer sciences and electronics, audiovisual and technologic equipment for teaching, high-quality media, etc. Quality is a determining factor for development that can attract investment (*the transparency of public institutions, stability, the predictability of policy, and rule of law and the regulatory environment*).

The local government determines the quantity of produced public goods by taking into consideration the quality financed by a "part" of the global tax revenues, noted $\varepsilon_i \in [0, 1]$ and $G_i + Q_i = t_i K_i + H_i \Rightarrow G_i = t_i K_i + H_i - Q_i$. However, the quality of the public goods is financed by a part ε_i of the global tax revenues giving us the function $Q_i(\varepsilon_i)$. Thus, the general form is $Q_i(\varepsilon_i) = \varepsilon_i [t_i K_i + H_i]$. Replacing Q_i by its value $\varepsilon_i [t_i K_i + H_i] \Rightarrow G_i = t_i K_i + H_i - Q_i$ the form of budget constraint local government, the quantity is the function the quality, where $G_i = (1 - \varepsilon_i) t_i K_i + H_i$. The local government can finance the provision of public goods regarding quantity quality in two ways. First, the local government finances the public good provision by

taxing the households (*lump-sum tax*) $dH_i \neq 0$, without varying their tax rate on capital, $dt_i = 0$. With respect to financing the quality of the public good provision ε_i , we have the following possibilities : (i) if $\varepsilon_i = 0$, this is interpreted by the idea that the local government provides its residents (households) with a public good in quantity G_i without quality Q_i with $G_i = t_i K_i + H_i$ (ii) if $\varepsilon_i \rightarrow 0$. This indicates that the government finances for quality Q_i of public goods as much as it does for their quantity G_i with $G_i = (1 - \varepsilon_i) [t_i K_i + H_i]$.

3. Objective of governments

The purpose of a local government is to maximize the social welfare of residents within its budget constraint. As the following program demonstrates:

$$\begin{cases} \max_{t_i, H_i} U_i = (C_i, G_i, Q_i) \\ B/C \mid C_i = F_i(K_i) - (\rho + t_i) K_i + \rho (\bar{K}/M) - H_i; & 1 - (a) \\ \mid G_i = (1 - \varepsilon_i) [t_i K_i + H_i] & 1 - (b) \end{cases} \quad (1)$$

The value $F_i(K_i) - (\rho + t_i) K_i$ corresponds to the land revenue paid by firms to lands owners. The amount $\rho (\bar{K}/M)$ refers to the return of capital invested by a resident regardless of his or her place of residence. Thus, the representative households in a local government only deduce the inhabitant tax from their revenues and allocate the rest to the consumption of the private good C_i . The condition of the first order gives the following equation.²

$$1 + MRS_{G_i, C_i} \frac{dG_i}{dC_i} + MRS_{Q_i, C_i} \frac{dQ_i}{dC_i} = 0 \quad (2)$$

, we begin by introducing the following definition of "the conventional rule of [Samuelson \(1954\)](#)". The Samuelson rule indicates that the sum of the "disposable" revenue to pay for a marginal increase of one unit of public good between private good MRS^i must be equal to the cost of the marginal unit of public good in terms of a private good. (i) If the sum of MRS^i is greater than the cost of an additional unit, households will be better off with one more unit of public good. (ii) If the sum of MRS^i is below the cost of an additional unit, they will be better off with one unit less than public good. Thus, the optimal provision of the allocation of public good can exist only if the Samuelson rule is observed. When the implanted tax is entirely "lump-sum tax", we must have the conventional rule of [Samuelson \(1954\)](#) characterizing the optimality of a pure public good, i.e., when $MRS^i = MRT^i = 1$.³

1. The public education services whose organization and operation are provided by the State are subject to the responsibilities within the jurisdiction contributing to the development of this public service. In this regard, the State requires that governments spend on educational quality. See the example provided by [Hoyt and Jensen \(2001\)](#) on quality of provision of public good.

2. See Appendix1 for the result of the equation (2).

3. The standard literature defines the inefficient provision of public goods as an allocation characterized by the inequality between the marginal rate of substitution MRS^i and the marginal rate of transformation $MRT^i = 1$.

3.1 Assumption and resolution of the model

Any government must take two fundamental decisions. The first one concerns the level of "provision of public goods" offered to the residents, and the second one applies to the level of "taxes" and to the mode of distribution of taxes between households and capital. We subsequently describe the social objectives of local governments in terms of supply of public goods, and we present a resolution of the model by giving different configurations of the public goods provision. According to the economic definition proposed by Samuelson (1954), public goods are necessary means in economic transactions among markets. What is the impact of improvement on quality of the public goods in economic transactions? The low quality of public goods causes deficiencies in supplying public-goods resulting in inefficiencies in markets in terms of productivity or transaction costs. Nonetheless, the high quality of public goods is an important factor in the economic development and attractiveness of the capital. From this perspective, the issue of the quality of the public goods is of paramount importance. The high quality of public goods also serves to enhance the legitimacy of governments, and as a consequence, it is an important factor revealing the preferences of people. Based on the equation (2), we should study three assumptions on the behavior of households. The first two assumptions indicate that households should have a preference for one of the substitutions appearing in equation (2): $MRS_{Q_i,C_i} = 0$, or MRS_{G_i,C_i} . The third assumption is a more global approach: $MRS_{G_i,C_i} \neq 0$ and $MRS_{Q_i,C_i} \neq 0$. We propose to formally present our assumptions before resolving each of these assumptions. The objective of the local government is to maximize the welfare of its residents, which solves the maximization program under the budget constraint.

- **Assumption A₁**

In assumption **A₁**, we assume that private goods and the quality of public goods are perfect complements. Hence, we have $MRS_{Q_i,C_i} = 0$. This behavior corresponds to expenses that the local government undertake for increasing the quality of public goods that is "latent".¹ The term latent is interpreted by the idea in which the quality of public good remains hidden but might become visible at some point during adaptation in supply of this specific good. Under this assumption, the equation (2) is $MRS_{G_i,C_i} = -dC_i/dG_i$. This is exactly the same result that we obtain in the context of fiscal competition with only private and public goods

1. See the report on economic freedom of the world. An example of a public good is the legal system. Without a good legal system, the economy cannot work efficiently. Thus, a good legal system complements the provision of private goods.

without quality. There are a plethora of firms that have specific needs for public goods. For instance, a transportation company needs high-quality roads, whereas a call center and employees working in both firms demand a good communication infrastructure; however, they are reluctant to pay extra money to ensure the quality. These examples are essential for the argumentation that the existence of latent quality lies in the underlying assumption of private goods and quality of public goods MRS_{G_i,C_i} we use **1 – (a)** and **1 – (b)** in the welfare of residents (households) program (1). By using the relation $MRS_{G_i,C_i} = -dC_i/dG_i$ and replacing dC_i and dG_i with its values, we obtain the following equation.

$$MRS_{G_i,C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (3)$$

- **Assumption A₂**

On the other hand, under the assumption **A₂**, we assume that private goods and public goods are perfect complements, $MRS_{G_i,C_i} = 0$ (given the same utility level, there is no exchange between quantity of private goods and the quantity of public goods). Consequently, according to equation (2), we obtain $MRS_{Q_i,C_i} = -dC_i/dQ_i$. The result of the second assumption indicates that the households are sensitive to the quality of the public goods, and they consider the expenses that the government pays for the quality of public goods. The quality of the public goods is no longer latent but influences the expenses undertaken on the quality of public goods. In this case, an improvement of the transportation network leads to employees' better efficiency and higher productivity. *Pari passu* improves efficiency of market transactions, and an effective legal system has positive impact on both firms and employees. By using assumption **A₂**, the government offers the quality of public good. By using the relation $MRS_{Q_i,C_i} = -dC_i/dQ_i$ and replacing dC_i and dQ_i with its values, we obtain the following equation.²

$$MRS_{Q_i,C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - dG_i} \quad (3a)$$

- **Assumption A₃**

In assumption **A₃**, we consider a more global approach where the households costlessly substitute private goods for public goods (quantity) according to $MRS_{G_i,C_i} = -dC_i/dG_i + MRS_{Q_i,C_i}(dQ_i/dG_i)$. The previous equation is comprised of two parts. First, the part related to the quantity of the public good referred to as $-dC_i/dG_i$. Second, the component related to the quality of contribution to the public good denoted as $MRS_{Q_i,C_i}(dQ_i/dG_i)$. According to the underlying assumption, the solution of model depends on the fiscal choice made by the government. The Assumptions **A₁** and **A₂** require the households have a preference for "substitutions" appearing in equation (2), that is, MRS_{G_i,C_i} or MRS_{Q_i,C_i}

2. See Appendix 2 for the result of the equation (3a).

the general case where the government provides the public good in terms of quantity and quality. Based on A_3 , we have the following equation.

$$MS_{G_i.C_i} = \frac{K_i dt_i + dH_i - MRS_{Q_i.C_i} [t_i dK_i + K_i dt_i + dH_i - dG_i]}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (3b)$$

In the following sections, our analysis will be based on the equations (2), (3), (3a), and (3b) to figure out the conditions in which Samuelson's rule is observed, and we propose different interpretations for each assumption.¹

3.2. Lump-Sum tax and provision of public goods

We consider that the local governments entirely finance the provision of public-goods to observe the standards of quality with a lump-sum tax $H_i \neq 0$, without taxation on capital income $dt_i = 0$.

Proposition 1. *When a local government uses a lump-sum tax $dH_i \neq 0$ to finance public goods regarding quantity and quality without any variation in the taxation on capital $dt_i = 0$, the Rule of Samuelson $MRS^i = MRT^i = 1$ is observed according to the following assumptions:*

- i. Under assumption 1, the public goods are optimal, $MRS_{G_i.C_i} = 1$ if and only if $d\varepsilon_i = 0$;
- ii. Under assumption 2, the quality of the public goods is optimal, $MRS_{Q_i.C_i} = 1$ is optimal if and only if the elasticity is equal $e_{G_i.H_i} = 0$;
- iii. Under assumption 3, the public -goods are optimal, $MRS_{G_i.C_i} = 1 - MRS_{Q_i.C_i}[\varepsilon_i]$ if and only if the variation of funding the quality is equal $d\varepsilon_i = 0$.

Proof. If the local government uses the lump-sum tax to finance the provision of public goods in terms of quantity and quality, it can increase the tax on households $dH_i \neq 0$ but cannot increase the tax on capital $dt_i = 0$. Based on A_1 , and using equation (3), we obtain the following equation $MRS_{G_i.C_i} = 1/1 - [\varepsilon_i]$ that we can rewrite as follows:

$$MRS_{Q_i.C_i} = \begin{cases} 1 & \text{if } d\varepsilon_i = 0; \\ 1/1 - [\varepsilon_i] & \text{if } 0 \leq d\varepsilon_i < 1; \\ +\infty & \text{if } d\varepsilon_i \rightarrow 1. \end{cases} \quad (4)$$

Based on equation (4), we have three possibilities for the provision of the public goods depending on ε_i if $d\varepsilon_i = 0$, meaning that local government provides a public good in quantity with a minimum quality ,and its level is optimal as indicated by the Samuelson's rule. This condition equalizes the marginal rate of substitution between the quantity of public and private goods and the marginal rate of transformation, that is, $MRS_{G_i.C_i} = MRT_{G_i.C_i} = 1$. In contrast, if $0 \leq d\varepsilon_i < 1$, $MRS_{G_i.C_i} = 1/1 - [\varepsilon_i] > 1$, and the provision of public goods is not optimal as the $MRS_{G_i.C_i} > 1$, and because the marginal cost is related to financing for the quality of the public goods. Finally, if $d\varepsilon_i$ tends toward unity and if $d\varepsilon_i \rightarrow 1$, the variation in choice of funding by the government is at its maximum level, so $\lim_{d\varepsilon_i \rightarrow 1} = +\infty$ that implies

economically that the two goods are perfect complements. Hence, we have the following result $MRS_{G_i.C_i} = +\infty$. based on assumption A_2 , equation (2), and equation (3a). The local government finances the quality of public good provision by taxing the households $dH_i \neq 0$ without varying the tax rate on capital, $dt_i = 0$ implying that $K_i = 0$. We obtain the form of $MRS_{Q_i.C_i} = dH_i/dH_i - dG_i$, so we can write the following equation $MRS_{Q_i.C_i} = 1/1 - dG_i/dH_i$ with $dG_i/dH_i = e_{G_i.H_i}$ representing the elasticity of quantity of public goods in relation to lump-sum tax. The form of $MRS_{Q_i.C_i} = 1/1 - e_{G_i.H_i}$ that we can rewrite is as follows:

$$MRS_{Q_i.C_i} = \begin{cases} 1 & \text{if } e_{G_i.H_i} = 0; \\ \frac{1}{1 - e_{G_i.H_i}} & \text{if } e_{G_i.H_i} \neq 1. \end{cases} \quad (4a)$$

According to the equation (4a), the provision of the quality of public goods depends essentially on the $e_{G_i.H_i}$. If the elasticity is equal to zero, $e_{G_i.H_i} = 0$. In this case, the optimum (if the tax lump-sum H_i is used to its maximum level H) level of quality of public good is achieved because this condition equalizes $MRS_{Q_i.C_i} = MRS_{G_i.C_i} = 1$. In contrast, if $e_{G_i.H_i} \neq 1$, the provision of public goods is not optimal because the $MRS_{G_i.C_i} > MRT_{G_i.C_i} = 1$ as funding the quality by government is at its maximum level, $MRS_{Q_i.C_i} > 1$. Based on the assumption A_3 and using equation (3b), we obtain the following equation.

$$MRS_{G_i.C_i} = \frac{1}{1 - [\varepsilon_i]} \{1 - MRS_{Q_i.C_i} \cdot \varepsilon_i\} \quad (4b)$$

We can rewrite the equation (4b), as follows:

$$MRS_{G_i.C_i} = \begin{cases} 1 - MRS_{Q_i.C_i} \cdot \varepsilon_i & \text{if } d\varepsilon_i = 0; \\ \frac{1}{1 - [\varepsilon_i]} & \text{if } 0 \leq d\varepsilon_i < 1; \\ +\infty & \text{if } d\varepsilon_i \rightarrow 1. \end{cases} \quad (4c)$$

Based on the equation (4c), the optimality of public goods depends on variation in funding the quality $d\varepsilon_i$.² If $d\varepsilon_i = 0$, $MRS_{G_i.C_i} = 1 - MRS_{Q_i.C_i} \cdot \varepsilon_i$ (the public goods provision is optimal). Then , $MRS_{G_i.C_i} = 1 - MRS_{Q_i.C_i}[\varepsilon_i]$ is equal to unity and less than the marginal contribution of the quality to public goods $MRS_{G_i.C_i}$. We recall that financing this quality can be determined by $0 \leq d\varepsilon_i < 1$. However, if $d\varepsilon_i \neq 0$, the choice of financing the quality by the government is deemed as ineffective. Consequently, the Samuelson's condition with a marginal cost of public funds is equal $1/1 - [\varepsilon_i] > 1$ and related to the financing the quantity. Finally, a value of $d\varepsilon_i$ which tends toward $d\varepsilon_i \rightarrow 1$ indicating that the financing

1. See Appendix 2 for the result of the equation (3b).

2. The optimality of public goods not only is a function of an increased funding but also depends on the transparency, competition, and a transparent bidding process. The quality of public goods also depends on the transparency of the bidding process.

this quality by the government tends to be at its maximum level.

The idea behind proposition 1 is that the government can achieve the optimum results if the tax lump-sum H_i is used at its maximum level \bar{H} . Based on the equation $C_i = F_i(K_i) - (\rho + t_i) K_i + \rho(\bar{K}/M) - H_i$, we will determine the reaction of the private consumption in the tax lump-sum: $\partial C_i / \partial H_i = -1 < 0$ and $\partial C_i / \partial \bar{H} = 0$ is affected by an increase in lump-sum tax H_i , and it will decrease due to an increase in the lump-sum tax to its maximum level $H_i \rightarrow \bar{H}$. These conditions decrease the welfare of individuals.

3.3. Tax on capital and provision of public Goods

We assume that the local government no longer has the possibility to finance the public goods by taxing households. The government finances the provision of the public goods by increasing tax rates on mobile capital there fore t_i . In this case (option-b), we offer the following proposition:

Proposition2. $dt_i \neq 0$ and dH_i if the local government increases the tax rates on mobile capital. If the assumptions A_1, A_2 and A_3 are met, Samuelson's rule is observed under certain conditions depending on e_{K_i,t_i} and ε_i :

- i. Under assumption 1, if $MRS_{G_i,C_i} = 1$, so $d\varepsilon_i = 1 - [K_i dt_i / e_{K_i,t_i} + 1]$;
- ii. Under assumption 2, the quality of the public goods is optimal $MRS_{Q_i,C_i} = 1$, so $\varepsilon_i = \Delta^{[K_i,t_i]}$;
- iii. Under assumption 1, the public goods are optimal $MRS_{G_i,C_i} = 1$ if and of only if $d\varepsilon_i = \frac{K_i}{2} dt_i [MRS_{Q_i,C_i}(\varepsilon_i + e_{K_i,t_i}) - 1]$.

Proof. $dt_i \neq 0$ and $dH_i = 0$. Hence, the local government cannot provide households with a sufficiently high level $dH_i = 0$, and in this case, it must increase tax rates on mobile capital. However, impose mobile capital instead of the immobile households generates a strong distortion because the government is subject to the double constraint, that is, financing the quality ε_i and lump-sum tax $dH_i = 0$. Under assumption A_1 and using the equation (3), we obtain the following form:¹

$$MRS_{G_i,C_i} = \frac{K_i dt_i}{[e_{K_i,t_i} + 1](1 - d\varepsilon_i)} MRT_{G_i,C_i} \quad (5)$$

According to equation (5), we need to consider two parameters. (i) The variation in financing the quality of public goods $d\varepsilon_i$; and(ii) the elasticity of capital to tax rates e_{K_i,t_i} . According to equation (5), if the $MRS_{G_i,C_i} = 1$, the variation in financing the quality $d\varepsilon_i$ is equal to the difference $(1 - [K_i dt_i / e_{K_i,t_i} + 1])$ between the marginal rate of transformation equal to unity. The term $(K_i dt_i / e_{K_i,t_i} + 1)$ measures the pressure that capital

market puts on decreasing tax rate on the capital, which impacts the efficiency of provision of public goods or the provision of public goods. Therefore, the following condition is necessary $d\varepsilon_i = (1 - [K_i dt_i / e_{K_i,t_i} + 1])$ with $K_i \neq 0, dt_i \neq 0$ and $\frac{1}{K_i dt_i} [e_{K_i,t_i} + 1] \neq 0$. This condition equalizes the marginal rate of substitution $MRS_{G_i,C_i} = 1$ and the marginal rate of transformation $MRT_{G_i,C_i} = 1$. The most plausible explanation is as follows: the adjustment of the funding of quality by the government is deemed as efficient in this case because it confirms the rule of Samuelson $MRS_{G_i,C_i} = MRT_{G_i,C_i} = 1$. In contrast, if $MRS_{G_i,C_i} \geq 1$, this implies that $d\varepsilon_i \geq (1 - [K_i dt_i / e_{K_i,t_i} + 1])$ financing the quality should be reduced because of "fiscal externality". Under assumption A_2 , the equation is as follow:²

$$MRS_{Q_i,C_i} = \frac{K_i dt_i}{e_{K_i,t_i} + \varepsilon_i} MRT_{Q_i,C_i} \quad (5a)$$

According to equation (5a), the marginal rate of substitution MRS_{Q_i,C_i} depends on ε_i and on the elasticity of capital to tax rate e_{K_i,t_i} . The analysis focuses on the budget ε_i (method of financing quality) that government decides to allocate to effectively finance the quality of the public goods. If the $MRS_{Q_i,C_i} = 1$, then $\varepsilon_i = [K_i dt_i - e_{K_i,t_i}]$ with $K_i dt_i$ the tax revenue of government i. This difference $K_i dt_i - e_{K_i,t_i}$ is equal to the tax revenue of government i which is less than the elasticity of the capital tax rate as we note $\Delta^{[K_i,t_i]} = K_i dt_i - e_{K_i,t_i}$. However, if $\varepsilon_i \leq \Delta^{[K_i,t_i]}$ (high elasticity e_{K_i,t_i} decreases, low elasticity e_{K_i,t_i} increases ε_i , respectively and consequently influences the provision of quality of the public goods). We can conclude that the government depends mainly on the elasticity e_{K_i,t_i} and to a lesser extent on ε_i to finance the quality. Under assumption A_3 and according to equation (3b), we obtain the following equation.³

$$MRS_{G_i,C_i} = \frac{K_i dt_i}{2[1 - d\varepsilon_i]} [1 - MRT_{Q_i,C_i}[\varepsilon_i + e_{K_i,t_i}]] \quad (5b)$$

Two explanations for the inefficiency of provision of quantity and quality of public goods can be identified when governments use tax on capital $dt_i \neq 0$ which is mobile. When they increase the tax on capital by one unit, or consequently a negative "fiscal externality": (i) the households will support an "additional cost" since there will be capital outflows to other governments, (ii) the loss of fiscal revenues results in a reduction of provision of public goods in terms of both quality and quantity. This tax variation $dt_i \neq 0$ must be high enough to not only pay for the marginal resource cost of provision of

1. See Appendix 3 for the result of equation (5).

2. See Appendix 4 for the result of equation (5a).

1. See Appendix 5 for the result of equation (5b).

public goods but also to offset the negative impact of the capital outflow on tax revenue. Equation (5b) is a generalized variant of the Samuelson's rule, which is a modified version of the rule. According to equation (5b), if $MRS_{Q_i, C_i} = 1$, we have $d\varepsilon_i = \frac{K_i}{2} dt_i [(\dots) - 1]$ or $d\varepsilon_i \geq \frac{K_i}{2} dt_i [(\dots) - 1]$.

However, for a value to correspond to the loss or gain in terms of capital, it should be equal to $|d\varepsilon_i|$ in absolute value. Consequently, the government decides either to lower or raise the quality of the public good as a function of quantity. Thus, we have the following statement confirming our analysis in terms of welfare. Based on equations (5), (5a) and (5b), the rate of capital taxation modifies the conditions of allocation of resources (in the sense of a larger choice in the provision of quality and quantity of public goods). If the elasticity e_{K_i, t_i} of capital tax rate is strong, the welfare in terms of the provision of public goods may decrease. Contrarily, if the elasticity e_{K_i, t_i} is low, the welfare increases. The elasticity terms explain the variation in financing the quality of public goods $d\varepsilon_i$ resulting from the adjustment of the rate of capital taxation $d\varepsilon_i \neq 0$ equalizing to the capital delocalized. With the equation of the private consumption $C_i = F_i(K_i) - (\rho + t_i) K_i + \rho(\bar{K}/M) - H_i$, we should determine the value of capital delocalized to other local governments

$$K_i = \frac{[C_i - F_i + H_i - \rho(\bar{K}/M)]}{\rho + t_i} < 0 \quad (6)$$

The variation of the capital to tax on capital rate (elasticity e_{K_i, t_i})

$$\frac{\partial K_i}{\partial t_i} = \frac{-K_i \rho + M[C_i + H_i - F_i]}{\rho + t_i} < 0 \quad (6a)$$

The variation of the private goods to tax on capital rate

$$\frac{\partial C_i}{\partial t_i} = -K_i < 0 \quad (6b)$$

Rate of tax on capital

$$t_i = \frac{K_i \rho - M[\rho - C_i - H_i - F_i]}{M} > 0 \quad (6c)$$

The impact of the variation of the tax on capital $dt_i \neq 0$ for the private goods C_i is as follows: the representative resident owns a fraction of the available capital stock K_i in the economy, and $dt_i \neq 0$ implies that the capital K_i is delocalized to other local governments leading to a drop in the production of private goods.

3.3. Comparison of the marginal cost of public funds

When the government increases the tax rates on mobile capital $dt_i \neq 0$ or raises taxes on households' lump-sum $dH_i \neq 0$ in order to increase the provision of public goods, there is a change in allocation of resources usually resulting in losses of efficiency of "public goods" in the economy. The cost of taxes

paid by the private sector is in general higher than the fiscal revenues perceived by the governments due to the loss of efficacy related to taxation. This loss of efficacy results from an increase on the tax rate on capital and can be easily measured by the marginal cost of public funds. The marginal cost of public funds (MCPF) measures the loss for a firm when the government increases the fiscal revenue by one monetary unit. For example, if the tax rate increases with $dt_i = 10\%$, and firms react by reducing the tax by 2%, the fiscal revenues received by the government increases by 8% rather than 10%.

The MCPF can be measured under assumption A_3 for the two approaches (tax on capital and lump-sum). We define the MCPF concept, which is derived from the model proposed by Atkinson and Stern (1974) in which a single government uses a distorting tax on production factors (inputs). Atkinson and Stern (1974) demonstrated that the Samuelson's (1954) rule for the optimum provision of public goods needs to be modified to account for tax distortions. The optimal level of provision of public goods should be lower if the marginal cost of public funds is higher.¹

Definition 1. A marginal cost of public funds measures the ratio between the marginal social cost of collecting additional resources referred to as ω and the social marginal value of private income denoted as β , that is, $MCPF = \frac{\omega}{\beta}$.

The purpose is to compare the marginal cost of public funds between the tax on capital and the lump-sum tax approaches in order to identify the best welfare. We shall verify whether the result provided by the MCPF ratio corresponds to the above-mentioned analysis based on assumption A_3 . When considering definition 2, the marginal cost of public funds corresponds under A_3 with $dH_i \neq 0$ denoted $MCPF^{H_i}$ and under A_3 when $dt_i \neq 0$ denoted $MCPF^{t_i}$. We provide the following proposition:²

Proposition 3. For a marginal cost of public funds MCPF, if assumption A_3 holds, we have the following assertions:

- i. if $dH_i \neq 0$ and $dt_i = 0$, so the marginal cost of public funds is equal to $MCPF^{H_i} = 1/[1 - d\varepsilon_i]$;
- ii. if $dH_i = 0$ and $dt_i \neq 0$, the marginal cost of public funds is equal to $MCPF^{t_i} = \frac{K_i dt_i}{2[1 - d\varepsilon_i]}$;
- iii. If $K_i dt_i / 2 < [1 - d\varepsilon_i]$, this implies that the $MCPF^{H_i} < MCPF^{t_i}$, so the welfare is better when the public goods are financed by the lump-sum tax. In contrast, if $K_i dt_i / 2 > [1 - d\varepsilon_i]$, we have the $MCPF^{H_i} > MCPF^{t_i}$, so it is better to finance welfare by the tax rates on mobile capital.

1. For more details on this question, see the studies by Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), Ahmad and Stern (1984), Wildasin (1984), Mayshar (1991), Ahmed and Croushore (1995), Snow and Warren (1996), Dahlby (1998), and Sandmo (1998).

2. See Appendix 6 for the result of proposition 3.

Proof. To determine the marginal cost of public funds for both cases under the general assumption A_3 , we will use equations (4a) and (5b). According to equation (4), we have $MRS_{G_i,C_i} = 1/[1 - d\varepsilon_i][1 - MRS_{Q_i,C_i} \times [\varepsilon_i]]$ allowing us to determine $MCPF^{H_i}$ as follows:

$$MRS_{G_i,C_i} = MCPF^{H_i} = 1/[1 - MRS_{Q_i,C_i} \times [\varepsilon_i]] \quad (7)$$

Based on equation (7), the marginal cost of public funds is $MCPF^{H_i} = 1/[1 - d\varepsilon_i] \geq 1$ if $d\varepsilon_i$ i.e., the funding of the quality is at its optimum level and the provision of public goods is optimal because $MRS_{G_i,C_i} = [1 - MRS_{Q_i,C_i} \times [\varepsilon_i]]$ or the MRS_{G_i,C_i} is equal to unity less the marginal contribution of the quality to public goods equal to $MRS_{Q_i,C_i} \times [\varepsilon_i]$. However, if $d\varepsilon_i \neq 0$ then $MCPF^{H_i} > 1$, the provision of public goods are suboptimal $MRS_{G_i,C_i} > 1$ if and only if, $MCPF^{H_i} > 1/[1 - MRS_{Q_i,C_i} \times [\varepsilon_i]]$, and this is valid only if $1 - MRS_{Q_i,C_i} \times [\varepsilon_i] \neq 0$ which occurs when $MRS_{Q_i,C_i} > 1$ ($MRS_{Q_i,C_i} \neq \frac{1}{\varepsilon_i}$). We can conclude that the funding of the quality of the public goods is the reason for inefficiency. On the other hand, according to equation (5b), the marginal cost of public funds can be found through the MRS_{G_i,C_i} by using the following equation where the $MCPF^{t_i} = \frac{K_i dt_i}{2[1 - d\varepsilon_i]}$.

$$MRS_{G_i,C_i} = MCPF^{t_i} [1 - MRS_{Q_i,C_i} [\varepsilon_i + e_{K_i,t_i}]] \quad (8)$$

The marginal cost of public funds reflects the distortionary effects of raising the marginal tax rate on capital $dt_i \neq 0$. This modification of the Samuelson's rule focuses only on the distortionary effects raising from increasing the tax rate on capital and financing of the quality ε_i . If the equation (8) : $MRS_{G_i,C_i} > 1$ if and only if the following conditions are met: $MCPF^{t_i} > 1$, it is necessary that the $MRS_{Q_i,C_i} < 0$ and $-1 \leq e_{K_i,t_i} \leq 0$ with $e_{K_i,t_i} < \varepsilon_i$. Comparing welfare of individuals in the two approaches, we know that the $MCPF^{H_i} = 1/[1 - d\varepsilon_i]$ with the variation of funding the quality $0 \leq d\varepsilon_i < 1$ that implies that the marginal cost of public funds, $MCPF^{H_i} > 1$. In contrast, the marginal cost of public fund when $dt_i \neq 0$ is equal to $MCPF^{t_i} = \frac{K_i dt_i}{2[1 - d\varepsilon_i]}$. In this case, we have two configurations depending on $K_i dt_i$ if $K_i dt_i > 2[1 - d\varepsilon_i]$ implying that the $MCPF^{t_i} > 1$ with $(K_i dt_i < 2[1 - d\varepsilon_i]) \Rightarrow MCPF^{t_i} < 1$. The comparison of the marginal cost of public funds indicates that if $K_i dt_i / 2 < [1 - d\varepsilon_i] \Rightarrow MCPF^{H_i} < MCPF^{t_i}$ or $K_i dt_i / 2 > [1 - d\varepsilon_i] \Rightarrow MCPF^{H_i} > MCPF^{t_i}$.

The classic approach provides the most general answer to the question of the optimal public goods supply in an economy with distortionary taxation. The sub-optimality of the provision of public goods is related to distortionary taxation. The local governments have a poor estimate of the marginal cost of public funds. Firstly, this distortion is related to the mobility of capital e_{K_i,t_i} following with an increase in the tax $dt_i \neq 0$. Secondly, it results from the loss of capital $-K_i$ causing a reduction in

References

government fiscal revenues and a decrease in funding the quality of the public goods.

4. Conclusion

We suggested a model to investigate the behavior of local governments facing a financial choice in terms of quality of the public goods. More specifically, we examined whether it is optimal for governments to tax the mobile capital, and under which circumstance, the Samuelson's condition for optimal allocation of resources between the private good and the public good can be met. The model suggested in the present study is aimed at providing suggestions on how to finance the supply of public goods given some required standards on its quality. A comparison of the results of different approaches confirm that when the public goods are financed by the lump-sum tax, the inefficiency of the provision of public goods arises from the mode of financing the quality of the public goods, and it does not occur from the choice of household taxation (lump-sum tax). Moreover, the findings demonstrated that taxation of mobile capital generates a strong distortion because the government is subject to the double constraint that is, financing of the quality and the elasticity of tax on capital. The impact of the variation of tax on capital on private goods is as follows: if the government increases the tax $dt_i \neq 0$, the capital will locate in other governments, and this relocation decreases the tax revenue for the government. The results in terms of welfare depend on the marginal cost of public funds. First, when the government uses a lump-sum tax to finance the quantity and quality of public goods, welfare is considered to improve as the marginal cost of public funds is lower compared to the second option where $dt_i \neq 0$ results in a higher marginal cost of public funding. When local governments decide not to tax capital, the optimal supply of public goods is less constrained by the funding choice of the quality (immobile households). These results appear to be crucial and strategic for the achievement of any optimum target. When capital is taxed, the Samuelson's condition is modified due to the existence of an additional cost born by households and due to a distorting tax. Nonetheless, our model suffers from a number of limitations. Is it realistic to assume that the only alternative to a tax on capital is a lump sum tax? Is it useful to compensate through an income or VAT tax? Is there any difference if a local tax is used to finance the quality of public goods or if the same local tax is imposed to finance a higher quantity of the public goods? Specification of the utility and the production functions are required to determine whether the results of our analysis will be affected.

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Appendices

Appendix 1

To resolve the program of maximization for a households, we assume the following method:

$$\Leftrightarrow U_i(C_i, G_i, Q_i) = 0 \tag{1}$$

$$\Leftrightarrow \frac{U_{C_i}dC_i}{U_{C_i}dC_i} + \frac{U_{G_i}dG_i}{U_{C_i}dC_i} + \frac{U_{Q_i}dQ_i}{U_{C_i}dC_i} = 0 \tag{2}$$

$$\Leftrightarrow 1 + \frac{U_{G_i}dG_i}{U_{C_i}dC_i} + \frac{U_{Q_i}dQ_i}{U_{C_i}dC_i} = 0 \tag{3}$$

$$\Leftrightarrow 1 + MRS_{G_i,C_i} \frac{dG_i}{dC_i} + MRS_{Q_i,C_i} \frac{dQ_i}{dC_i} \tag{4}$$

With $\frac{U_{C_i}dC_i}{U_{C_i}dC_i} = MRS_{G_i,C_i} \frac{dG_i}{dC_i}, \frac{U_{Q_i}dQ_i}{U_{C_i}dC_i} = MRS_{Q_i,C_i} \frac{dQ_i}{dC_i}$

•Assumption A₁

By using assumption A_1 , we calculate the marginal rate of substitution between the quantity of private-good and the quantity of the public-goods

$$\left\{ \begin{array}{l} \max_{t_i H_i} U_i = (C_i, G_i, Q_i) \\ B/C \mid C_i = F_i(K_i) - (\rho + t_i) K_i + \rho \left(\frac{\bar{K}}{M} \right) - H_i \quad 1 - (a) \\ \mid G_i = (1 - \varepsilon_i) [t_i K_i + H_i] \quad 1 - (b) \end{array} \right. \quad (1)$$

We use 1-(a) and 1-(b) in the welfare of residents (households) program (4a). By using the relation $MRS_{G_i, C_i} = -\frac{dC_i}{dG_i}$ and replacing dC_i and dG_i by its values, we obtain

$$MRS_{G_i, C_i} = \frac{F_{K_i} dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (5)$$

The arbitrage condition equals the net return of capital in each local government $\rho = F_{K_i} - t_i$ with ρ is the net return of the capital and t_i is the tax on mobile capital. We substitute the return of net capital condition, ρ , in (5) and we obtain

$$MRS_{G_i, C_i} = \frac{\rho dK_i + t_i dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (5a)$$

After simplification we have

$$MRS_{G_i, C_i} = \frac{-\rho dK_i - t_i dK_i + \rho dK_i + t_i dK_i + K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (5)$$

And we obtain

$$MRS_{G_i, C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (5c)$$

Appendix 2

Assumption A_2

We use 1-(a) and 1-(b) in the welfare of residents (households) program (4a). The local government determines the quantity of public goods produced by taking into consideration the quality financed with a "part" of the global tax revenues, noted ε_i with $\varepsilon_i \in [0, 1]$ and $G_i + Q_i = t_i K_i + H_i - G_i$ By using the relation $MRS_{Q_i, C_i} = -\frac{dC_i}{dQ_i}$ and replacing dC_i and dQ_i by its values, we obtain

$$MRS_{Q_i, C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - dG_i} \quad (5d)$$

From equation (4), which defines the program of maximization for a households, we assume the following relation:

$$MRS_{G_i, C_i} \frac{dC_i}{dG_i} = -1 - MRS_{Q_i, C_i} \frac{dC_i}{dQ_i} \quad (6)$$

$$\Rightarrow MRS_{G_i, C_i} = -\left[\frac{dC_i}{dG_i} \right] + MRS_{Q_i, C_i} \left[\frac{dQ_i}{dG_i} \right] \quad (7)$$

$$MRS_{G_i, C_i} = \frac{dC_i + MRS_{Q_i, C_i} [dQ_i]}{dG_i} \quad (7a)$$

Assumption A_3

Under A_3 , and replacing dC_i , dG_i and dQ_i by its values on equation (7), we obtain the following equation that represents the general case

$$MRS_{G_i, C_i} = \frac{K_i dt_i + dH_i - MRS_{Q_i, C_i} t_i [t_i dK_i + K_i dt_i + dH_i]}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (8)$$

Appendix 3

Assumption A_1 ,

The government funds the provision of the public-goods by a tax on mobile capital therefore $dt_i \neq 0$, and $dH_i = 0$ under A_1 the calculation the marginal rate of substitution between the private good and the quantity of the public good MRS_{G_i, C_i} requires the budget constraint:

$$\left\{ \begin{array}{l} \max_{t_i H_i} U_i = (C_i, G_i, Q_i) \quad (8a) \\ B/C \mid C_i = F_i(K_i) - (\rho + t_i) K_i + \rho \left(\frac{\bar{K}}{M} \right) - H_i \quad 1 - (a) \\ \mid G_i = (1 - \varepsilon_i) [t_i K_i + H_i] \quad 1 - (b) \end{array} \right.$$

By using the formula $MRS_{G_i, C_i} = -\frac{dC_i}{dG_i}$ and replacing dC_i , dG_i by its values, we obtain:

$$MRS_{G_i, C_i} = \frac{F_{K_i} dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (9)$$

We substitute the capital net return condition $F_{K_i} - t_i = \rho$ in (9) and we obtain

$$MRS_{G_i, C_i} = \frac{\rho dK_i + t_i dK_i - \rho dK_i - t_i dK_i - K_i dt_i - dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (10)$$

After simplification we have

$$MRS_{G_i, C_i} = \frac{-\rho dK_i - t_i dK_i + \rho dK_i + t_i dK_i + K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (11)$$

$$MRS_{G_i, C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad ((12))$$

With $dH_i = 0$ we obtain

$$MRS_{G_i, C_i} = \frac{K_i dt_i}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (13)$$

We divide by

$$\frac{MRS_{G_i, C_i}}{K_i dt_i} = \frac{\frac{K_i dt_i}{K_i dt_i}}{\frac{t_i dK_i + K_i dt_i - d\varepsilon_i [t_i dK_i + K_i dt_i]}{K_i dt_i}} \quad (14)$$

$$\frac{MRS_{G_i, C_i}}{K_i dt_i} = \frac{1}{e_{K_i, t_i} + 1 - d\varepsilon_i [e_{K_i, t_i} + 1]} \quad (15)$$

The party $e_{K_i, t_i} + 1 - d\varepsilon_i [e_{K_i, t_i} + 1]$ is written $[e_{K_i, t_i} + 1](1 - d\varepsilon_i)$ we obtain

$$\frac{MRS_{G_i, C_i}}{K_i dt_i} = \frac{1}{[e_{K_i, t_i} + 1](1 - d\varepsilon_i)} \quad (16)$$

$$MRS_{G_i, C_i} = K_i dt_i \left[\frac{1}{[e_{K_i, t_i} + 1](1 - d\varepsilon_i)} \right] \quad (16a)$$

$$\text{If } MRS_{G_i, C_i} = 1 \Rightarrow 1 = K_i dt_i \left[\frac{1}{[e_{K_i, t_i} + 1](1 - d\varepsilon_i)} \right] \quad (16b)$$

Solution is:

$$d\varepsilon_i = 1 - \frac{K_i dt_i}{e_{K_i,t_i} + 1} \quad (16c)$$

With $K_i \neq 0, t_i \neq 0$ and $\frac{1}{K_i dt_i} (e_{K_i,t_i} + 1) \neq 0$

Appendix 4

Assumption A₂

We therefore consider that the private good C_i and the quantity of the public good G_i are of the goods of perfect complement $MRS_{G_i,C_i} = 0$ under assumption A₂,

$$\text{the } MRS_{Q_i,C_i} = -\frac{dC_i}{dQ_i} \\ MRS_{Q_i,C_i} = \frac{K_i dt_i + dH_i}{t_i dK_i + K_i dt_i + dH_i - dG_i} \quad (16d)$$

With $dt_i \neq 0, dK_i \neq 0$ and $dH_i = 0$ we obtain

$$MRS_{Q_i,C_i} = \frac{K_i dt_i}{t_i dK_i + K_i dt_i - dG_i} \quad (16e)$$

By dividing on $K_i dt_i$ we obtain

$$\frac{MRS_{Q_i,C_i}}{K_i dt_i} = \frac{1}{\frac{t_i dK_i}{K_i dt_i} + 1 - \frac{dG_i}{K_i dt_i}} \quad (16f)$$

With $\frac{t_i dK_i}{K_i dt_i} = e_{K_i,t_i}$ and $\frac{dG_i}{dt_i} = -K_i(\varepsilon_i - 1) < 0$ by replacing the latter value in equation (16f), we obtain

$$\frac{MRS_{Q_i,C_i}}{K_i dt_i} = \frac{1}{\varepsilon_i + e_{K_i,t_i}} \quad (17)$$

$$MRS_{Q_i,C_i} = K_i dt_i \left[\frac{1}{\varepsilon_i + e_{K_i,t_i}} \right] \quad (17a)$$

$$\text{If } MRS_{Q_i,C_i} = 1 \Rightarrow 1 = K_i dt_i \left[\frac{1}{\varepsilon_i + e_{K_i,t_i}} \right] \Rightarrow \varepsilon_i = K_i dt_i - e_{K_i,t_i} \quad (17b)$$

Appendix 5

Assumption A₃

Under A₃, the following equation that represents the general case, we obtain

$$MRS_{G_i,C_i} = -\frac{dC_i}{dG_i} + MRS_{Q_i,C_i} \frac{dQ_i}{dG_i} \quad (18)$$

Replacing dC_i, dQ_i and dG_i by its values, we obtain

$$MRS_{G_i,C_i} = \frac{K_i dt_i + dH_i - MRS_{Q_i,C_i} [t_i dK_i + K_i dt_i + dH_i - dG_i]}{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]} \quad (18a)$$

We divide by $K_i dt_i$ the two parts of the equation (18a)

$$\frac{MRS_{G_i,C_i}}{K_i dt_i} = \frac{\frac{K_i dt_i + dH_i}{K_i dt_i} - MRS_{Q_i,C_i} \left[\frac{t_i dK_i + K_i dt_i + dH_i - dG_i}{K_i dt_i} \right]}{\frac{t_i dK_i + K_i dt_i + dH_i - d\varepsilon_i [t_i dK_i + K_i dt_i + dH_i]}{K_i dt_i}} \quad (19)$$

$$\frac{MRS_{G_i,C_i}}{K_i dt_i} = \frac{1 + \frac{dH_i}{K_i dt_i} - MRS_{Q_i,C_i} \left[e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} \right]}{e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} - d\varepsilon_i \left[e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} \right]} \quad (20)$$

$$\frac{MRS_{G_i,C_i}}{K_i dt_i} = \frac{1 + \frac{dH_i}{K_i dt_i} - MRS_{Q_i,C_i} \left[e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} \right] - \frac{dG_i}{K_i dt_i}}{e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} - d\varepsilon_i \left[e_{K_i,t_i} + 1 + \frac{dH_i}{K_i dt_i} \right]} \quad (20a)$$

With $\frac{dG_i}{dt_i} = -K_i(\varepsilon_i - 1) < 0, \frac{dH_i}{dt_i} = 0$

$$\frac{MRS_{G_i,C_i}}{K_i dt_i} = \frac{1 - MRS_{Q_i,C_i} \left[e_{K_i,t_i} + 1 - \frac{1}{K_i} - K_i(\varepsilon_i - 1) \right]}{e_{K_i,t_i} + 1 - d\varepsilon_i [e_{K_i,t_i} + 1]} \quad (21)$$

The party $e_{K_i,t_i} + 1 - d\varepsilon_i [e_{K_i,t_i} + 1]$ is written $[e_{K_i,t_i} + 1](\varepsilon_i - 1)$ after simplification, we obtain

$$\frac{MRS_{G_i,C_i}}{K_i dt_i} = \frac{1 - MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i)}{[e_{K_i,t_i} + 1](\varepsilon_i - 1)} \quad (21a)$$

Which gives us

$$MRS_{G_i,C_i} = K_i dt_i \left[\frac{1 - MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i)}{[e_{K_i,t_i} + 1](\varepsilon_i - 1)} \right] \quad (21b)$$

We can rewrite equation (21b) as follows

$$MRS_{G_i,C_i} = K_i dt_i \left[\frac{1 - MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i)}{(\varepsilon_i - 1)} \right] \frac{K_i dt_i}{[e_{K_i,t_i} + 1]} \quad (22)$$

With $e_{K_i,t_i} = \frac{t_i dK_i}{K_i dt_i}$ we can rewrite equation (22) as follows

$$MRS_{G_i,C_i} = K_i dt_i \left[\frac{1 - MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i)}{(\varepsilon_i - 1)} \right] \frac{K_i dt_i}{\left[\frac{t_i dK_i}{K_i dt_i} + 1 \right]} \quad (23)$$

With $\frac{K_i dt_i}{\left[\frac{t_i dK_i}{K_i dt_i} + 1 \right]} = \frac{1}{2} K_i dt_i$

$$MRS_{G_i,C_i} = [1 - MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i)] \frac{K_i dt_i}{2(1 - d\varepsilon_i)} \quad (24)$$

With $MRS_{G_i,C_i} = 1$ solution is

$$d\varepsilon_i = \frac{K_i dt_i}{2} [MRS_{Q_i,C_i} (e_{K_i,t_i} + \varepsilon_i) - 1] + 1 \quad (25)$$

With $-K_i dt_i (MRS_{Q_i,C_i} \times (e_{K_i,t_i} + \varepsilon_i) - 1) \neq 0$

Appendix 6

From the standard MCPF definition, and without any variation in the taxation $dt_i = 0$, the optimal rule for public-goods provision are characterized by

$$MRS_{G_i,C_i} = \frac{1}{(1 - d\varepsilon_i)} \{1 - MRS_{Q_i,C_i} \times \varepsilon_i\} \quad (26)$$

$$\Leftrightarrow MRS_{G_i,C_i} = MCPF^{Hi} \{1 - MRS_{Q_i,C_i} \times \varepsilon_i\} \quad (27)$$

By

$$MCPF^{Hi} = \frac{1}{(1 - d\varepsilon_i)} > 1 \quad (28)$$

The government funds the provision of the public good by a tax on mobile capital therefore $dt_i \neq 0$ the optimal rule for public-goods provision are characterized

$$MRS_{G_i,C_i} = \frac{K_i dt_i}{2(1 - d\varepsilon_i)} \{1 - MRS_{Q_i,C_i} \times (\varepsilon_i + e_{K_i,t_i})\} \quad (29)$$

$$MRS_{G_i,C_i} = MCPF^{ti} \{1 - MRS_{Q_i,C_i} \times (\varepsilon_i + e_{K_i,t_i})\} \quad (30)$$

By

$$MCPF^{ti} = \frac{K_i dt_i}{2(1 - d\varepsilon_i)} > 1 \quad (31)$$

Under the two generals approaches, we have the following result: (a) under A₃ with $dH_i \neq 0$, the financing of the quality of the public good ε_i is the reason for the effectiveness of the public good; (b) under A₃ with $dt_i \neq 0$, the tax rate $dt_i \neq 0$ and the financing of the quality of the public good ε_i are the reasons for inefficiency the provision of public good.

