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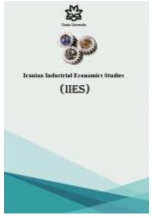

Application of distance function and data envelopment analysis in efficiency evaluation of Islamic Azad University branches in East Azerbaijan province

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ABSTRACT

The main aim of the study was to evaluate the efficiency of Islamic Azad University branches in East Azerbaijan using stochastic frontier analysis (SFA) and to compare the findings with the results of data envelopment analysis (DEA). In this respect, SFA and DEA techniques were applied to measure production efficiency of 28 branches of East Azerbaijan Islamic Azad University considering the following variables as either output or input: 3 variables as output (including the number of graduated students and students accepted for the further pursuit of their studies, as well as the number of articles, books, and research projects) and 6 variables as input (including the number of university faculty members, educational fields, current students, staff working in different sections of the university, and the total university campus in square meters as well as university costs). Estimation of DEA and obtaining of the units' efficiency indicated that the average efficiency of the universities under investigation were 0.66 and 0.80 in SFA and DEA techniques, respectively. In addition, efficiency distributions in SFA and DEA methods were tremendously different from each other so that in SFA method nearly 40% while in DEA method more than 60% of unit efficiency was between 0.8-1 intervals. The minimum level of estimated efficiency (0.14) was observed for Hadishahr using SFA method, and in DEA approach, the minimum level of efficiency belonged to Khoda Afarin branch (0.065). Finally, considering that non-parametric DEA tends to represent the units more efficiently and thus a decrease in the difference among the estimated efficiency values of the units, it is recommended that university authorities apply parametric SFA for performance evaluation and identification of less efficient units.

1. Introduction

Efficiency has been seriously discussed among economists since the beginning of the 1950s and new methods have been practically established regarding efficiency measurement. Early efforts regarding the investigation of efficiency concept and its measurement were made by Koopmans (1951) and Debreu (1951) who both examined merely technical efficiency. In pursuit of them, Farell (1957), proposing a method for the application of isoquant production curves and minimizing the use of production inputs, was the first one to empirically measure efficiency. He introduced his theories regarding efficiency measurement in an article entitled "*The Measurement o Productive Efficiency*" for the first time and empirically analyzed the issues relating the efficiency measurement in agriculture section of America for a set of productive units observed.

Following the presentation of a theoretical framework by Farrell regarding efficiency measurement, Aigner et al, (1977) in their study, *formulation and estimation of stochastic frontier production function models*, practically measured efficiency according to Farrell's definition and using stochastic frontier production function method. The motive for the formation of this model stemmed from the idea that deviations from frontier production may not be under the control of decision-maker (production) units and the effects of which should be estimated. In fact, Aigner et al., introducing mixed error term in non-parametric models (a part of which indicates inefficiency and another part includes stochastic disturbances in the model), provided the possibility of various statistic inferences about efficiency estimation and also more precise analysis of frontier functions.

On the other hand, Charnes et al (1978) suggested a new idea regarding the measurement of efficiency and non-parametric method (linear programming). They added general data analysis method, which is based on mathematical programming techniques, to economic literature to extend Farrell's method in a way to include the process of multiple productive factors and multiple output case as well. An article entitled "*Measuring the efficiency of decision-making units*" by Charnes et al (1978) along with Aigner et al.'s (1977) work are actually recognized as two main articles to suggest two noble and classic methods (parametric and non-parametric) regarding efficiency measurement methods. The models proposed in many other articles were generally based on one of the two methods introduced in the abovementioned articles.

Generally, efficiency is one of the most important criteria being less focused on by internal organizations and firms. In other words, lack of complete knowledge on efficiency and finally efficiency of the organization are of the main issues of current organizations. According to Ganley and Cubbin viewpoints (1992), investigation of the internal performance of every organization is carried out due to several main reasons as follows:

- In order to manage the work of units under their control, organizations should explore those units' situations through appropriate criteria.
- To create a sense of responsibility in executive managers, their performance is rewarded by punishment and encouragement system and in doing so the context for competition among the units is provided.
- To obtain appropriate criteria for allocation of budget and available resources among the units through performance investigation.

As stated by Ganley and Cubbin (1992), high-level managers who are responsible for total budget allocation across various organizations can become aware of the situation of that organization through investigating the general performance of each organization's units and thus use it as an appropriate criterion for allocating the budget. Meanwhile, university chancellors of Islamic Azad University units (as higher education centers) also need to determine the situation and state of their units. As previously mentioned, SFA approach is one of the most important methods applied in assessing the efficiency of economic units and in case there exists more than one output, distance function method is used to measure production efficiency. Considering the fact that the university is regarded as an organization which produces various outputs in contrast to the application of different inputs, this study aims to evaluate the efficiency of Islamic Azad University units using distance function. In addition, the important issue in the present study is that whether the efficiency evaluation of units under investigation would yield different results using parametric (distance function) or non-parametric (DEA) methods.

Organization of the paper is as follows:

Following the first section, namely, the introduction, the related literature is reviewed in the second section, and in the third section, research models and variables are provided. Empirical findings as well as results are dealt with in section four. Finally, conclusions of the study are presented in the fifth section.

2. Review of the literature

In this section of the study, first, theoretical frameworks regarding efficiency measurement are provided and then a few empirical and important studies are briefly explained.

2.1. Theoretical framework

Generally speaking, efficiency, as stated by Shafiee (2017), is a criterion for evaluating the performance of a system from different aspects. It indicates that how a firm or an organization has used its available resources to achieve the most optimal production over a period of time. According to him, with regard to the fact that a variety of factors play a role in increasing the efficiency level, one can estimate and assess the performance through determining and estimating the efficiency and also identifying the importance of each factor in determining the amount of output. Shafiee (2017) believed that the efficiency of each unit is derived from the comparison of the unit's indices with standards and since the standard for indices can be within or outside the society, then efficiency is defined as absolute or relative.

The basis for efficiency measurement and productivity studies was first begun by Farrell in 1957. His significant role regarding the new insight is more obvious in two areas: how to define efficiency and productivity and also how to use technology benchmarking and to compute efficiency. In Farrell's method, inefficiency is defined as the distance of each firm from the production function of the firm being accepted as the model. As stated by Farrell, the base for this measurement is as a radial structure that includes the distance from the observed (unobserved) inefficient point to the reference point on production frontier. In case the real production point of the firm is on frontier point, then the decision maker unit is completely efficient; however, if it is under the frontier point, it is regarded as inefficient.

In Farrell et al.' viewpoints, efficiency is of different kinds including technical, allocative, economic, as well as structural efficacy. Technical efficiency, according to Mo'meni and Shahkhah (2009), demonstrates the ability of a firm to obtain maximum level of output from a series of given inputs. In addition, allocative efficacy shows the ability of a firm in using the inputs of optimum ratios considering contradictory costs of the inputs as well. As stated by them, economic efficacy is a combination of technical as well as allocative efficiency. When an organization is efficient in terms of technique and allocation, it is said that the organization has economic efficiency. And finally, Mo'meni and Shahkhah defined structural efficacy as a type of efficiency that can be used to evaluate the efficiency of the industry. Structural efficiency of an industry is obtained by the weighted average efficiency of that industry's firms (Hadian, E., & Azimi Hosseini, A, 2004).

The rate of efficiency, according to Shafiee (2017), is a value between 0-1. It implies that if the value is 1, then the decision maker unit is completely efficient. For instance, if the measured efficiency is 90% in contrast to cost frontier, it means that the decision maker unit can decrease its costs up to 10% without any change in outputs. Farrell (as cited in Shafiee, 2017) introduced non-parametric methods of measuring efficiency for the first time and practically calculated the efficiency of agriculture section of America.

2.2 Empirical studies

Various studies have been conducted regarding the evaluation of efficiency using different methods particularly SFA, some of which are provided in the following sections.

Shafiee (2017) proposed a multi-stage data envelopment analysis and investigated the efficiency of 10 branches of an Iranian bank. The results indicated that all the branches, which according to Charnes, Cooper, and Rhodes (CCR) model, had an efficiency value of 1, did not necessarily have an efficiency value of one in the new model; since, according to the new model, if the organizational unit has a perfect cooperation at leader-member level, it is said that the organization's efficiency level equals to one and that both (leader, member) have an efficiency level of one.

Alirezaee and et al (2016) investigated the efficiency level of 5 selected insurance companies applying two-phase DEA and window analysis models. Using the data related to companies' performance during the period 2010 to 2013, they showed that firms' inefficiency during these periods is related to the weakness in the second phase meaning that insurance firms often act weakly in profitability phase. In addition, the results of the study also indicated that there exists a significant gap between the initial and optimal intermediate measures.

In their study, Shoja and Darvish Motevali (2015), measured and compared the performance of research department of 14 selected branches of Islamic Azad University during the period 2010-2013 using a DEA model without input. The results indicated that in the first year, 4 university units of Takestan, Karaj, Firouzkouh, and Varamin were efficient. In addition, in the second year, university units of Parand, Takestan, Roudehen, Karaj, and Firouzkouh were efficient as well. And finally, 4 branches, namely, Parand, Takestan, Qazvin, and Firouzkouh were highly efficient in the third year.

Safdari Ranjbar and et al, (2013) evaluated the efficiency of 15 engineering departments of Amirkabir University of Technology using DEA. The model used for DEA was a constant return to scale. The results showed that faculties of Mathematics, Chemistry, and Computer Sciences were regarded as the most efficient faculties. In contrast, faculties of Aerospace, Shipbuilding, and Textile Engineering were more inefficient than other faculties.

Mashayekhi and Shafipour (2012) investigated the efficiency of accounting education system in universities of Iran. DEA approach was used in this respect. The results revealed that in terms of resource usage, the most efficient accounting departments were within the faculties of Economics and

Management and the least efficient ones were Human Sciences faculties.

In another study by Alam Tabriz, and et al (2010), the efficiency of Shahid Beheshti University faculties was explored using an integrated approach of DEA and goal programming models. The findings demonstrated that faculties of Science, Management and Accounting, and Earth Science were the most efficient faculties. The faculty of Law was also reported as the least efficient faculty in periods of 2004-2007.

Using the stochastic frontier approach of research effort cost, Entezari (2008) attempted to measure and analyze the economic efficiency of knowledge firms (independent research and development units). The application of this approach for knowledge firms of Iran showed that presence of economic efficiency in knowledge firms could be highly confirmed. In addition, the results revealed that some characteristics of knowledge firms such as ownership, size, and research collaboration with other organization affect the efficiency of knowledge firms.

Similarly, Entezari and Arabmazar Yazdi (2007) explored the production function of knowledge firms and external factors affecting its efficiency using SFA production model. The findings of the study indicated that in contrast to industrial firms, the production process in knowledge firm consisted of two stages. In the first stage, new explicit knowledge was produced through research while, in the second stage, it was merged with tactic knowledge and translated into knowledge products leading to commercialization. In both stages, the decreasing returns were dominant to scale. Unexpectedly, there was no inefficiency in explicit knowledge production stage. However, inefficiency was high in knowledge conversion stage.

Duma and Kasman (2018) examined environmental technical efficiency for European Union (EU) member countries in the period 1990–2011 using parametric distance function. They also assessed the environmental technical efficiency convergence among the sampled countries. The results indicated that environmental technical efficiency scores varied among countries under investigation and EU-15 countries, compared to new members and the candidate countries, had a greater potential for reducing CO2 emissions while increasing gross domestic product (GDP). In addition, there existed environmental technical efficiency convergence among EU sample countries.

Yamori and et al., (2017) investigated the efficiency of Japanese financial cooperatives applying a parametric distance function approach. Using monthly data of financial cooperatives during 2009-2014, they indicated that local cooperatives that had experienced drastic consolidation over the past two decades and increased their size were more efficient than the other groups.

Similarly, Sagarra et al., (2017) carried out a research in order to explore the efficiency of Mexican universities using data envelopment analysis. Using the data from 55 universities over the period of 2007-2012, they demonstrated that efficiency rate in universities under investigation varied a lot.

Assessing the efficiency of Italian public universities over a three year period (2008–2011) using parametric distance function, Agasisti, T.,Barra et al (2016) found that models which were estimated without considering unobservable heterogeneity yielded divergent efficiency estimates. Besides, considering differences in universities' performances, according to geographical areas, and estimating the efficiency of universities, appropriate state-level policies could be suggested.

In addition, in another investigation, Mikusova (2015) evaluated the efficiency of the Czech public universities applying DEA and the university data from 2013. Moreira Lopez et al., (2006) in their study used stochastic production frontiers (SPF) and stochastic distance frontiers (SDF) to measure technical efficiency of dairy farms in an area in Argentina. This research was carried out using panel data including 46 observations from 1997/98 to 2001/02. The results of the study showed that the average efficiency of the firms ranged from 67.2% to 88.4%.

In their study on estimating the efficiency level of Portuguese public universities, Afonso and Santos (2005) found that if the model under investigation is input-oriented then the total estimated efficiency ranges from 0.55% to 0.68%. In addition, if it is output-oriented, the total efficiency of the universities would be between 0.73% and 0.83%.

Mizala et al (2002) examined the technical efficiency of schools in Chile. SPF and DEA were used to measure the efficiency. A sample of 2000 schools in Chile was investigated and both approaches yielded the same results regarding the efficiency of these schools. It implies that based on SPF and DEA, technical efficiency of schools in Chile were estimated 93% and 95%, respectively.

3. Research population, variables, and model

The population of the study included 28 branches of Islamic Azad University of East Azerbaijan Province the efficiency of which was investigated in 2015. The related data regarding the university units are presented in Table 1. The unit of Tabriz University was removed because of it inclusiveness and drastic difference compared to other university units.

Table 1

List of Islamic Azad University branches in East Azerbaijan Province

1	Tasoj	8	Varzqan	15	Bostan Abad	22	Marand
2	Hadishahr	9	Heris	16	Jolfa	23	Ahar

3	Torkmanchai	10	Hashtrood	17	Sarab	24	Bonab
4	Zonoz	11	Sofiyan	18	Shabestar	25	Osku
5	Kharvana	12	Khamene	19	Ajabshir	26	Maraghe
6	Horand	13	Mamqan	20	Kaleybar	27	Miyaneh
7	KHoda Afarin	14	Ilkhiji	21	Malekan	28	Azarshahr

The required statistics and data relating to units of Islamic Azad University of Azerbaijan were collected from instruction, research, and development sections through documentary method. Then, the data were

y_1: Students accepted for the further pursuit of their studies

y_2: Number of provided papers, books, research projects, and conferences, and

y_3: Number of graduate students.

Inputs are also as follows:

x_1: Number of faculty members,

x_2: Number of instructional fields,

x_3: Entire university campus in square meter,

x_4: Number of currently studying students,

x_5: Number of university staff working in different sections of the university, and

x_6: Costs of the university in 1000 Rials.

According to the data provided in literature review section and the above-mentioned variables, a research model which is an indicator of the stipulation of Cobb–Douglas stochastic output distance frontier is as follows:

$$-\ln y_{1i} = \beta_0 + \beta_1 \ln(\frac{y_{2i}}{y_{1i}}) + \beta_3 \ln(\frac{y_{3i}}{y_{1i}}) + \alpha_1 \ln x_{1i} + \alpha_2 \ln x_{2i} + \alpha_3 \ln x_{3i}$$
$$+ \alpha_4 \ln x_{4i} + \alpha_5 \ln x_{5i} + \alpha_6 \ln x_{6i} + v_i + u_i \qquad , \quad i = 1, 2, 3, ..., 28$$

Variables of the above model were previously defined. The variable *i* also represents university units. Considering the main aim of the study, namely, calculating the performance of university units by distance function as well as stochastic frontier and DEA models, in the following sections distance function is discussed and its estimation approach is explored. It is noteworthy to mention that the research model is calculated using maximum likelihood estimation (MLE) and Frontier software, version 4.1

Production technology can be represented in various ways. In this respect, the application of production, profit, cost, and even revenue functions are among the ways recommended in the literature on economics.

Distance function is another way to be used in representing production technology. This method is an indicator of production activity distance from production possibility frontier (PPF) and thus has a close relationship with technical efficiency

summarized and classified and the outputs and inputs of university units were determined as follows.

The outputs for university units

measurement. In fact, as stated by Kumbhakar and Lovell (2000), when price information is not available, technical efficiency is the only efficient criterion that can be measured by distance function. According to them, when more than one input is used to produce more than one outputs, then, Shephard's distance functions (1953, 1970) can describe the structure of production technology without a need to determine behavioral goals including cost minimization or profit maximization. Therefore, in these conditions, distance functions can be used to evaluate efficiency.

In order to define output-distance function, we consider a set of outputs *P(x)* which represent the likelihood of producing all the output vectors $y \in R_+^M$ using the input vector $x \in R_+^N$. That is:

$$P(x) = \{ y \in R_+^M \ : x \, can \, produce \, y \} \quad (2)$$

In this case, output distance function is defined according to output set *P(x)* as follows:

$$D_O(x, y) = \min \{ \theta : (y/\theta) \in P(x) \} \quad (3)$$

According to Kumbhakar and Lovell (2000), output distance function yields the minimum amount through which an output vector can be moderated and meanwhile produced by the available output vector. To better understand the concept of distance function, the following figures, which are depicted for one-input and one-output (Figure 1) and two-output and multi-input states (Figure 2), can be used.

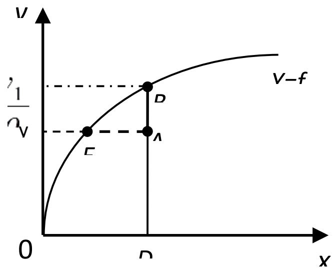

Figure 1. Distance Function for one-input

In Figure 1, production function *y=f(x)* is related to one-input and one-output state and is depicted under the assumption of decreasing returns to scale. The value of distance function equals the distance of each production point which is under production possibility curve. For instance, the value of distance function for a firm in point *A* equals to the ratio $\theta = DA/DB$. This value is equal to the reverse ratio through which the given firm can increase its outputs and continue using the primary input without a need to increase the amount of input *x*.

The above-mentioned ratio, as stated by Coelli and Perelman (1996), is exactly interpreted as technical efficiency as well. That is

$$(Technical\ Efficiency)\ TE = \frac{observed\ output}{frontier\ output}$$

$$= \frac{y}{f(x)} = \frac{DA}{DB} = \theta \ (Dista.nce\ Measure) \quad (4)$$

Generally, to study the efficiency, the production function can be represented as:

$$y_i = f(x_i).TE_i \quad \Rightarrow \quad \ln y_i = \ln f(x_i) + \ln TE_i \quad (5)$$

Using econometric models is one of the standard methods in efficiency studies. In case of one input and one output with the assumption that production function is of Cobb–Douglas type, then we can write equation (5) to estimate the form of econometric models in a way to have

$$\ln y_i = \beta_0 + \beta_1 \ln x_i + \varepsilon_i \quad , \quad \varepsilon_i = v_i - u_i \quad (6)$$

where ε_i is part of a blend and consists of two components, namely, v_i which is part a random error and u_i that is an indicator of technical inefficiency. Considering the issue that the statements $lnTE_i$ and u_i represent firm efficiency in equations (5) and (6), respectively, therefore we will have the following equation:

$$\ln TE_i = -u_i \quad \Rightarrow \quad TE_i = \exp(-u_i) \quad (7)$$

comparing the relations (3) and (6), it can be seen that in the simple form of production function with a product, the result of statement $exp(-u_i)$ is equal to ϑ, the value of distance function that meanwhile can be considered as technical efficiency of the firm as well.

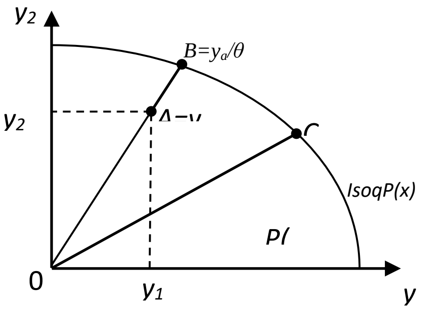

Figure 2 Distance Function for the firm that produces

Figure (2) shows a condition where two products of y_1 and y_2 are produced by input vector *x*. The value of distance function for the firm that produces an output vector comparable to point *A*, is equivalent to the ratio $\theta = 0A/0B$. Therefore, for a firm that is in point *A*, the ratio is $D_O(x, y) = \theta \ < 1$.

Euclidean distances of *OA* and *OB* are as follows:

$$OA = \|y_a\| = \sqrt{y_{1a}^2 + y_{2a}^2} \ , \qquad OB = \|y_b\| = \sqrt{y_{1b}^2 + y_{2b}^2}$$

Besides, it can be observed that the value of distance function for the points (points *B* and *C*) set on production possibility frontier, equals 1. Accordingly, distance function $D_O(x, y)$ adopts a value less than or equal to 1 if the output vector *y* is on production possibility frontier set (PPS). That is $D_O(x, y) \le 1 \ if \ \ y \in P(x)$. In addition, if *y* is placed on production possibility frontier set, then distance function would adopt a single value, namely,

$$D(x, y) = 1 \ if \ \ y \in Isoq\,P(x)$$
$$= \{y : y \in P(x), \omega y \notin P(x), \omega > 1\} \quad (8)$$

Furthermore, as Lovell et al., (1994) pointed out, distance function $D_O(x, y)$ has some characteristics that are directly driven from assumptions relating to technology set. In the following section, some of these characteristics are provided.

i. $D_O(x, y)$ should equal to zero for all non-negative values of x,
ii. $D_O(x, y)$ is non-decreasing to *y* and non-increasing to *x*,
iii. $D_O(x, y)$ is linear homogenous to *y*,
iv. $D_O(x, y)$ is quasi convex to *x*, and convex to *y*,
v. If *y* is a member of *x* output set ($y \in P(x)$), then, $D_O(x, y) \le 1$, and
vi. $D_O(x, y)$ equals 1 if *y* is a member of production possibility frontier set.

In the present study, stochastic frontier production was used to estimate distance function and also to evaluate efficiency level of units under investigation. Selection of an appropriate incidental form is one of the main issues to be considered in parametric empirical studies. Cobb-Douglas incidental form is one of the most common forms that is used in

$$-\ln y_i = \beta_0 + \sum_{m=1}^{M-1} \beta_m \ln \frac{y_{mi}}{y_{1i}} + \sum_{k=1}^{K} \beta_k \ln x_{ki} + v_i + u_i \quad (14)$$

empirical studies. According to Coelli and Perelman's study (1996), the Cobb-Douglas output distance frontier (D^O) could be written as:

$$\ln D_i^O(x, y) = \beta_0 + \sum_{m=1}^{M} \beta_m \ln y_{mi} + \sum_{k=1}^{K} \beta_k \ln x_{ki} \qquad (9)$$

where y_m and x_k which are used by firm *i* denote level of output *m* and also the value for input *k* , respectively. Lovell et al., (1995) indicated that stochastic distance frontier equation must fulfill the following conditions: symmetry, monotonicity, positive linear homogeneity, non-decreasing, and convex based on output (y), and decreasing according to inputs (x). The convexity condition is important to ensure that the distance frontier demonstrates diminishing marginal rates of technical substitution.

The homogeneity restriction, according to Lovell et al (1994), could be empirically imposed by normalizing all outputs in the function toward an arbitrary output (e.g., y_M). In other words, a suitable approach to imposing homogeneity restriction on equation (8) is to consider that homogeneity indicates that:

$$D_O(x,\omega y)=\omega D_O(x,y),\ for\, any\, \omega>0$$

Therefore, if an output (such as output M) is chosen arbitrarily and if $\omega=1/y_M$, then we will have:

$$D_O(x,y/y_M)=D_O(x,y)/y_M$$

As a result, the following statement is obtained for Cobb-Douglas form:

$$\ln(D_i^O(x,y)/y_{Mi})=\beta_0+\sum_{m=1}^{M-1}\beta_m\ln\frac{y_{mi}}{y_{Mi}}+\sum_{k=1}^{K}\beta_k\ln x_{ki}(10)$$

As can be seen in the above equation, if $y_m=y_M$, then the ratio of $\frac{y_{mi}}{y_{Mi}}$ equals to 1 and thus, its logarithm is equal to zero. Consequently, the summation in these statements is up to *M-1* and not *M*.

An appropriate approach must be chosen in order to obtain the estimations for unknown function parameters through selecting a suitable incidental form.

To this end, through algebraic simplification, we can write equation (10) as follows:

$$\ln(D_{Oi}/y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta),\ \ i=1,2,....,N(10)$$

$$\ln(D_{Oi})-\ln(y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta),\ \ i=1,2,....,N$$

Therefore, the statement can be written as:

$$-\ln(y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta)-\ln(D_{Oi}),\ \ i=1,2,....,N(12)$$

or

$$-\ln(y_{Mi})=TL(x_i,y_i/y_{Mi},\alpha,\beta)+v_i+u_i,\ \ i=1,2,....,N(13)$$

Considering the above explanation, equation (10) can be rewritten as:

As can be seen in equation (14), in stochastic frontier model which was introduced by Aigner et al., (1977), the specified frontier function has a two-part fault-component: one part is as symmetrical fault-component considered for statistical errors (vi) and the other part as asymmetrical fault-component considered for calculating inefficiency (ui which is obtained from a change in variable -ln(Doi)). Considering the appropriate assumptions regarding the type of vi and ui, parameters of Cobb-Douglas stochastic distance function can be estimated using maximum likelihood method.

The estimation of equation (14) by maximum likelihood method, according to Moreira Lopez et al., (2006), would produce unbiased parameters and efficient estimates for the stochastic output distance frontier.

4. Results

In this section, the research model is estimated and the efficiency is calculated for each of the university units as well. The results obtained from the estimation of model (14) through maximum likelihood method are provided in Table 2.

Table 2
Cobb-Douglas stochastic distance function estimation using maximum likelihood method

Variable	Coefficient	t-statistic
lny1	---	---
intercept	0.43	0.45
Ln(y2/y1)	0.49	3.96
Ln(y3/y1)	0.10	12.27
Lnx1	0.49	3.44
Lnx2	0.21	2.83
Lnx3	-0.18	1.07
Lnx4	0.64	5.40
Lnx5	0.07	1.12
Lnx6	0.27	2.34

sigma-squared = 0.649 (t-ratio= 5.58)
Log likelihood function = -14.137
Likelihood-ratio test of one-sided error (χ^2) = 23.65
prob($\geq\chi^2$) = 0.000
Number of Observations = 28

The estimation results of Cobb-Douglas output distance function in Table 2 shows that except for the ratios related to university campus and the number of staff inputs, other inputs' ratios reached a statistically significant level (p≤ .05). When interpreting the estimation results of the ratios, attention must be paid to the issue that the interpretation of ratios in one-output production function is different from that of multi-output production function (similar to distance function of the current study). In one-output production function (Cobb-Douglas type), the estimated ratios indicate production elasticity to the relevant input while in the present study, for example, the ratio lnx_1 with a value of 0.49 is an indicator of production elasticity of y_1 (output-dependent variable) to input x_1 by keeping the percentage stable for each output component (y_1, y_2, y_3) and also keeping the other outputs stable.

The computed efficiency index estimated by stochastic frontier for each university unit is provided in Table 3. As the results show, the range of efficiency changes for units of Islamic Azad University of East Azerbaijan Province are between -0.14 and 0.99. The minimum level of efficiency belonged to Hadishahr with an efficiency level of 0.14. Units of Ahar, Jolfa, Kaleybar, and Khoda Afarin were among the most efficient university units. The average efficiency of 28 university units was 0.66 as well.

Table 3

The estimated Efficiency Index for Each University Branch

Tasoj	0.58	Varzqan	0.24	Bostan Abad	0.54	Marand	0.72
Hadishahr	0.14	Heris	0.47	Jolfa	0.99	Ahar	0.99
Torkmanchai	0.16	Hashtrood	0.92	Sarab	0.90	Bonab	0.86
Zonoz	0.30	Sofiyan	0.46	Shabestar	0.69	Osku	0.60
Kharvana	0.99	Khamene	0.43	Ajabshir	0.70	Maraghe	0.99
Horand	0.94	Mamqan	0.67	Kaleybar	0.99	Miyaneh	0.78
KHoda Afarin	0.99	Ilkhiji	0.25	Malekan	0.25	Azarshahr	0.99

According to the efficiency index level, units of Ahar, Bonab, Maragheh, Azarshahr, Jolfa, Sarab, Kaleybar, Hashtrud, Kharvana, Hoorand, and Khoda Afarin were classified as units with high levels of efficiency. In contrast, units of Malekan, Varzaqan, Ilkhchi, Hadishahr, Torkamanchay, and Zonuz were regarded as units with low levels of efficiency. Other units had a moderate level of efficiency. In order to further investigate the issue, the results of the data envelopment analysis are reported. Meanwhile, the results of the estimation obtained through SFA are compared with the results of the estimations from DEA method in the following paragraphs.

The results related to the calculation of efficiency for each university unit under variable returns to scale (VRS) and constant returns to scale (CRS) conditions are provided in Table 4.

Table 4

The Estimated Efficiency Index for Each University Branch Based on DEA

Branch	**Efficiency Index**		**Branch**	**Efficiency Index**		**Branch**	**Efficiency Index**	
	CRS	**VRS**		**CRS**	**VRS**		**CRS**	**VRS**
Tasoj	0.418	0.499	Sofiyan	1	1	Malekan	1	1
Hadishahr	1	1	Khamene	1	1	Marand	0.798	0.798
Torkmanchai	1	1	Mamqan	0.967	0.967	Ahar	0.855	0.855
Zonoz	0.838	0.838	Ilkhiji	1	1	Bonab	0.958	0.958
Kharvana	0.339	0.421	Bostan Abad	1	1	Osku	1	1
Horand	0.343	0.539	Jolfa	1	1	Maraghe	0.641	0.642
Khoda Afarin	0.065	0.122	Sarab	0.499	0.847	Miyaneh	0.701	0.729
Varzqan	1	1	Shabestar	1	1	Azarshahr	1	1
Heris	1	1	Ajabshir	0.678	0.678			
Hashtrood	0.485	0.793	Kaleybar	1	1			

As can be seen in the above table, when DEA model is used to estimate the efficiency level of the units under investigation, the efficiency level of most units tends toward 1 (namely, perfect efficiency). This does not allow us to investigate the efficiency difference across the units more precisely. The comparison results of efficiency estimation of units under investigation (through SFA and DEA models) are provided in Table 5.

Table 5

Relative Frequency Distribution of Estimated Efficiency Based on SFA and DEA

Range of Estimated Efficiency	**Relative Frequency**		
	DEA		SFA
	CRS	VRS	
0-0.2	3.6	3.6	7.1
0.2-0.4	7.1	0.0	14.3
0.4-0.6	10.7	10.7	17.9
0.6-0.8	14.3	17.9	21.4
0.8-1.0	64.3	67.9	39.3

Mean	0.81	0.85	0.66

According to Table 5, the efficiency of the sample units in DEA method, under both conditions of VRS and CRS, was estimated more than that of in SFA. That is, the average efficiency of selected universities in SFA model was equal to 0.66 and more than 0.80 in DEA model.

Examination of the relative frequency distribution of estimated efficiency indicated that in all the three reported estimations (SFA, VRS, and CRS), the efficiency level of most universities was at 0.8-1 interval. Efficiency distribution in SFA method, though had a tremendous difference with estimated efficiency distribution in DEA model so as in SFA method around 40% and in DEA more than 60% of the units' efficiency was between 0.8-1 intervals. A more precious investigation suggested that the estimated efficiency distribution had a left-skewness in all the three states but the severity of skewness was different for the distributions. The ratios of skewness for the estimated efficiency distributions in three conditions of SFA, CRS, and VRS were equal to -0.84, -1.99, and -2.61, respectively. This implies that in the samples under investigation, first, the results obtained from DEA method tended toward showing more efficiency for the units than those from SFA method and second, the severity of this tendency in VRS method was more than CRS.

5. Conclusion

The main aim of the study was to evaluate the efficiency level of Islamic Azad University Units of East Azerbaijan Province using stochastic frontier analysis of production. In this respect, following determining the outputs and inputs of university units (including 28 units), stochastic output distance functions were specified for them using Cobb-Douglas incidental form.

The estimation of research model by MLE and obtaining the efficiency of the units showed that efficiency change of the units ranged from 0.14 to 1 and their average efficiency was 0.66 as well. According to the estimated efficiency for the sample units, Ahar, Bonab, Maragheh, Azarshahr, Jolfa, Sarab, Kaleybar, Hashtrud, Kharvana, Hoorand, and Khoda Afarin were found to be among the units with high efficiency. In addition, to further explore the issue, the estimations obtained from SFA of production and DEA were compared. The results revealed that efficiency level of units under investigation through DEA, under both conditions of VRS and CRS methods, were estimated to be higher than when it was explored through SFA of production.

Of the foregoing discussion on the issue, it can be concluded that university authorities must pay attention to the fact that efficiency estimation of different units is sensitive to the method used and that various methods can yield different results regarding the efficiency or lack of efficiency of a unit. In addition, since using the results of a non-parametric model of DEA tends to represent higher efficiency for the units and thus a decrease in the difference among the estimated efficiency values of the units, it is recommended that university authorities use SFA parametric model for efficiency evaluation as well as identification of less efficient units. It is obvious that if units with less efficiency are identified appropriately, suitable decisions can be made in order to improve the efficiency level of such units.

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